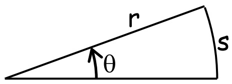
Essential Physics Knowledge … a work in progress!

STUFF EVERY PHYSICIST SHOULD KNOW.

Basics

$s = r\theta$



$v = \dfrac{ds}{dt}$ $\quad a = \dfrac{dv}{dt}$

- derivatives give v, ω, a, α and their relationships.

Notation: $\dot{y} = \dfrac{dy}{dt}$ and $y'(x) = \dfrac{dy}{dx}$

Wavelength and Wave Number: $k = \dfrac{2\pi}{\lambda}$

Magnitude of a complex number: |z| = z*z where z* is the complex conjugate of z

$\left|e^{i\phi}\right| = 1$

Vector dot product multiplies parallel components $\vec{a}\cdot\vec{b} = ab\cos\theta$ (scaler product)

Vector cross product multiplies perpendicular components $\left|\vec{a}\times\vec{b}\right| = ab\sin\theta$, dir. by RHR (vector product)

Momentum

Momentum: $\vec{p} = m\vec{v}$

De Broglie: $p = \dfrac{h}{\lambda} = \hbar k$

Angular Momentum: $\vec{L} = \vec{r}\times\vec{p} = rp\sin(\theta)$

- For r perpendicular to p (circular motion) $L = rp$

Energy

Work: W = (force)(distance in direction of force), $W_{s_1 \text{ to } s_2} = \int_{s_1}^{s_2} \vec{F}\bullet d\vec{s}$

Kinetic Energy: $K = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$

Potential Energy: $U_{s1 \text{ to } s2} = -W_{s1 \text{ to } s2}$ for conservative forces (W independent of path)

$$U_{s_1 \text{ to } s_2} = -\int_{s_1}^{s_2} \vec{F}_{conservative}\bullet d\vec{s}$$

Mechanical Energy: E = K + U

Work-Energy Theorem:

First form: $W_{total} = \Delta K$

Second form: $W_{non\text{-}conservative} = \Delta K + \Delta U$

Mechanics

Newton's Laws

1. Stuff coasts (it takes a force to change velocity magnitude or direction)
2. $\sum \vec{F}_{External} = m\vec{a}$
3. Stuff pushes back (forces act on two bodies equally in opposite directions)

Kinematics (only when $\vec{a}_0 = \text{constant}$**)**

$v(t) = v_0 + a_0 t$ no x
$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$ no v
} Derived from $\vec{a}_0 = \frac{d\vec{v}}{dt}$ and $\vec{v} = \frac{d\vec{x}}{dt}$

$v^2 = v_0^2 + 2a_0 (x - x_0)$ no t
$x = x_0 + \left(\frac{v + v_0}{2}\right) t$ no a
} Derived from first two equations by eliminating t or a

Electricity and Magnetism

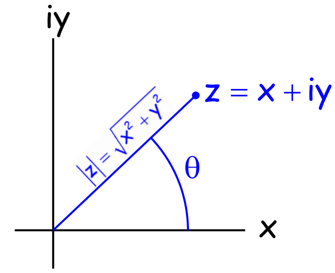
Force between point charges at distance r: $\vec{F} = \frac{kQq}{r^2}\hat{r} = \frac{Qq}{4\pi\epsilon_0 r^2}\hat{r}$

Force on a charge in a uniform Field E: $\vec{F} = q\vec{E}$

Electric Field = Force per charge: $\vec{E} = \frac{\vec{F}}{q}$

Electric Potential = Potential Energy per charge across L: $V = \frac{U}{q} = \frac{\vec{F} \bullet \vec{L}}{q} = \frac{q\vec{E} \bullet \vec{L}}{q} = EL$

Essential Modern Physics Knowledge

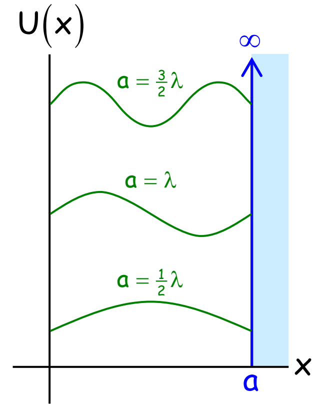


Complex Numbers

For $z = x + iy$, the complex conjugate is $z^* = x - iy$ and the absolute value is the distance on the complex plane from the origin to the point z.

$$|z|^2 = zz^* = (x+iy)(x-iy) = x^2 + y^2$$

We also utilize Euler's formula stating that $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$.



Particle in a Box: Allowed Energy States

For a quantum particle of energy, E, in a potential box where U(x) = 0 for 0 < x < a, it can only exist between x = 0 and x = a (since E = K + U and U = ∞ outside of 0 < x < a). **A quantum particle is described by a wave function** that can exist in the box at only certain wavelengths (λ, first three shown) the wave function must be zero at the sides (x = 0 & x = a).

For a standing wave, $\psi(x) = A\sin(kx) + B\cos(kx)$, this requires that

$ka = n\pi$ where n = 1, 2, 3, ... and $k = \frac{2\pi}{\lambda}$ is the wave number

**** Other than saying a quantum particle is described by a wave, this is just math! ****

The physics comes in with de Broglie:

$$p = \frac{h}{\lambda} = \hbar k \quad \text{for} \quad k = \frac{2\pi}{\lambda} \quad \text{and} \quad \hbar = \frac{h}{2\pi}$$

KNOW THESE RELATIONSHIPS!

Thus, the allowed momenta of the particle in the box are:

$$a = \frac{n}{2}\lambda = \frac{n\pi}{k} = \frac{n\pi\hbar}{p} \Rightarrow p = \frac{n\pi\hbar}{a}$$

BE ABLE TO FIGURE THESE OUT

Since the energy of the particle is purely kinetic (U = 0), $E = p^2/2m$ gives the allowed energies:

BE ABLE TO DERIVE THIS.

$$E = n^2 \frac{\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \ldots$$

TZDII (7.23)

Probability Density, Normalization, & Expectation Value

$$|\Psi(\vec{r},t)|^2 = \text{probability (volume) density for finding particle at } \vec{r}$$

TZDII (6.15)

Prob. of finding particle between x_1 & x_2 $= \int_{x_1}^{x_2} |\psi(x)|^2 dx \approx |\psi(x = x_1)|^2 \Delta x$

UNDERSTAND WHY INTEGRAL IS APPROXIMATED BY A PRODUCT.

Normalization

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

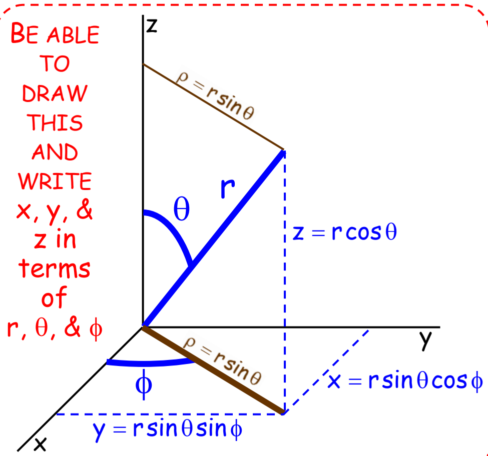
TZDII (7.55)

CARTESIAN COORDINATES GO FROM -∞ TO +∞

The expectation value (value expected after many measurements) of f(x) with a probability density $\left|\psi(x)\right|^2$ is

$$\int f(x)\left|\psi(x)\right|^2 dx = \int f(x)p(x)dx \qquad \text{TZDII (7.69)}$$

BE ABLE TO DRAW THIS AND WRITE x, y, & z in terms of r, θ, & ϕ



3-D SCHRÖDINGER EQUATION

For the hydrogen, we assume a purely radial potential due to the charge of the proton (PE = force x distance).

$$U(r) = -\frac{ke^2}{r}$$

The electron's energy can only be multiples of the Rydberg Energy as shown in TZDII equations 5.22 and 5.23.

$$E = -\frac{m_e\left(ke^2\right)^2}{2\hbar^2}\frac{1}{n^2} = -\frac{E_R}{n^2} = -\frac{13.6}{n^2}\text{eV}$$

KNOW THIS.

Separation of Variables

Assume that the wave function of the electron can be written as a product

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

Substituting this into the Schrödinger equation and setting

HAVE A CONCEPTUAL UNDERSTANDING OF THIS PROCESS.

$$\text{Function of } \phi = \text{Functions of } r \text{ and } \theta = -m^2$$

$$\text{Function of } \theta = \text{Function of } r = -k = -\ell(\ell+1)$$

Yields three differential equations, one in each variable that can be solved for various values of n, m and ℓ.

The ϕ and θ solutions depending on m and ℓ are the Spherical Harmonics with the θ solutions given in Table 8.1 as the Associated Legendre Functions.

The R solutions depending on n and ℓ are given in table 8.2. The normalization of these equations requires that the electron be found within a spherical volume, thus the differential volume element (supplied by the normalization of the θ and ϕ solutions in the full wave function) becomes $4\pi r^2 dr$, giving

$$\text{Prob. of finding } e^- \text{ in a spherical volume } = \int_{r_1}^{r_2} 4\pi r^2 \left|R(r)\right|^2 dr$$

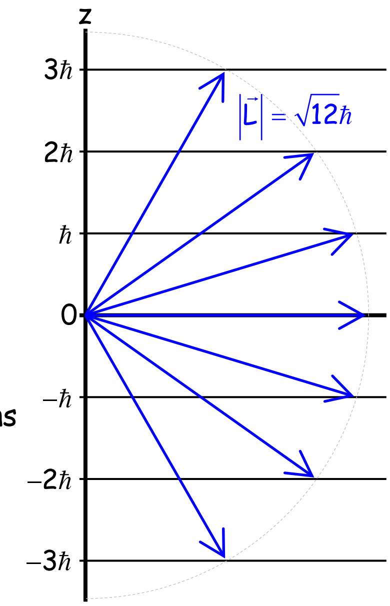
Normalization

$$\int_0^\infty 4\pi r^2 \left|R(r)\right|^2 dr = 1 \qquad \text{TZDII (8.84)}$$

r IN SPHERICAL GOES FROM 0 TO ∞

Know how to demonstrate a function is a solution to a differential equation.

Quantum Numbers give PHYSICAL QUANTITIES (that we can measure in the lab!)



Principle Quantum Number, n gives the energy of a state:

n = 1, 2, 3, …

$$E_n = -\frac{1}{2}\frac{m_e\left(ke^2\right)^2}{n^2\hbar^2} = \frac{-13.6\text{ eV}}{n^2}$$

Angular Momentum Quantum Number, ℓ gives magnitude of the angular momentum:

$$\left|\vec{L}\right| = \hbar\sqrt{\ell\left(\ell+1\right)}$$

ℓ = 0, 1, 2, 3, …, (n – 1)

The orbitals are named for the ℓ values,

s: ℓ = 0 = sharp
p: ℓ = 1 = principle
d: ℓ = 2 = diffuse
f: ℓ = 3 = fundamental

} Named for spectral line classifications

Magnetic Quantum Number, m gives the z-component of the angular momentum:

$$L_z = m\hbar$$

m = -ℓ, …, 0, … ,ℓ

Spin Quantum Number, m_s gives the z-component of the spin angular momentum:

$$S_z = m_s\hbar$$

$m_s = \pm\frac{1}{2}$

Magnitude of the Spin Angular Momentum is given by S and s:

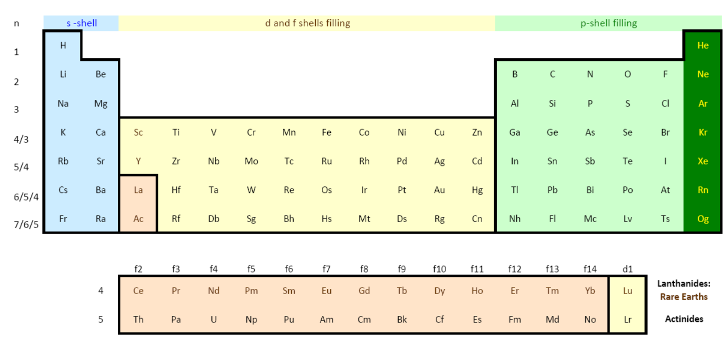
$$\left|\vec{S}\right| = \hbar\sqrt{s\left(s+1\right)} = \frac{\sqrt{3}}{2}\hbar,\quad s = \frac{1}{2}$$

Atoms and Molecules

As the number of protons increases in heavier nuclei, the electrons fill higher energy levels.

- The structure of the Periodic Table reflects the structure of electron energy levels

  s: 2e⁻, p: 6 e⁻, d: 10 e⁻, f: 14 e⁻



- electrons in inner shells shield nuclear charge from outer electrons (≲ 1 e of charge)
- electrons in the same shell shield charge from each other (≲ ½ e of charge)
- Elements with full shells (e.g. He, Ca, Zn, Kr) are the least chemically active in each row
- Elements one electron from full (Cl, Br) bind to fill that shell (high electron affinity)
- Elements with a single (or few) electrons in an outer shell (Na, Li) are easily ionized

STATISTICAL MECHANICS

Energy Distributions of Particles

To describe how the energy is distributed among particles in large collections (gases, liquids and solids), physicists developed different energy distributions based on the types of particles

$$n(E)dE = g(E)F_{\text{smart guys}}$$

→ $F_{\text{smart guys}}$: smart guy factor expressing behavior of particle type: classical, Fermion, or Boson

→ $g(E)$: density of states (number of E < available energy states < E + dE

→ $n(E)dE$: number of particles with E < energy < E + dE [n(E) = number/energy …number energy

Maxwell-Boltzmann Statistics: Classical Particles

Classical particles are distinguishable, only interact with each other through elastic collisions and are at a low enough density that the wave functions don't overlap.

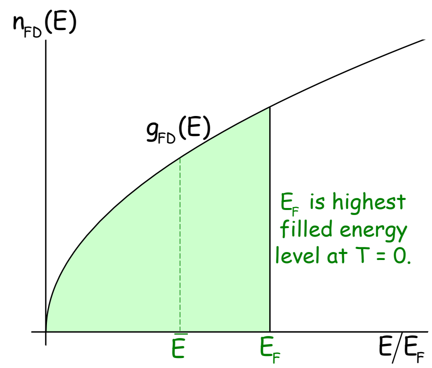
The mean square is the "average" physicists use since it relates to kinetic energy

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Classical vs. Quantum Statistics

Quantum statistics must be used if the particle wave functions overlap.

Fermi-Dirac Statistics: Fermion Quantum Particles



Fermions have $\frac{1}{2}$-integer spins & obey Pauli Exclusion Principal

Fermi Energy, E_F is the highest occupied state at T = 0 and

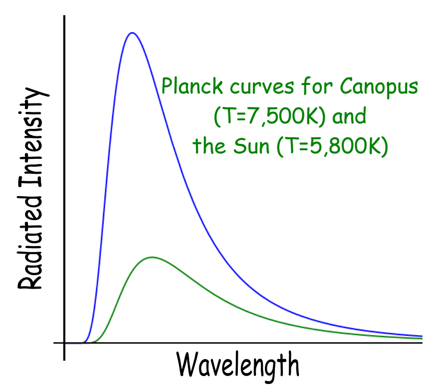
Fermi Temp. thermal energy equals Fermi: $E_F = kT_F$

Fermi Velocity $E_F = \frac{1}{2}m(v_F)^2$

Fermi-Dirac Factor: $$F_{FD} = \frac{1}{e^{(E-E_F/kT)}+1}$$

Why can we act as though T = 300K ≈ 0K?

Bose-Einstein Statistics: Boson Quantum Particles



Bosons have zero or integer spins and do not obey the Pauli

Bose-Einstein Factor $$F_{BE} = \frac{1}{e^{E/kT}-1}$$

Why is normalization factor set equal to one?

Planck Curve

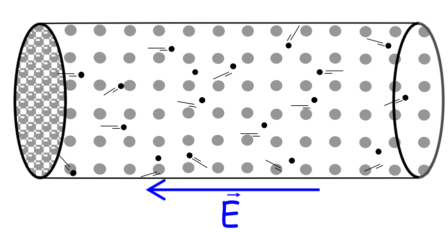
What situation did Planck imagine to derive curve?

What situation did Bose imagine to derive curve?

Wien Displacment Law $\lambda_{max}T = \text{constant}$

Stefan Boltzmann Law $R(T) = \epsilon\sigma T^4$

Electrical Conductivity



Ohm's Law: $V = RI$ or $\vec{J} = \sigma\vec{E}$

How do Drude's and Einstein's models differ?

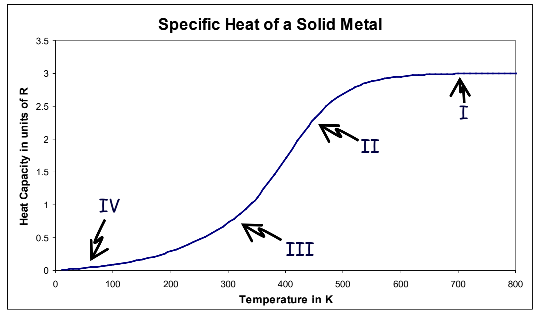
Heat Capacity

Into what bins does equipartition sort energy to explain the curve shown?

Which region(s) explained by MB statistics

Which region(s) explained by FD statistics

Which region(s) explained by BE statistics



Nuclear Physics

Nuclei are protons (p^+) and neutrons (n^0) held together by Strong Nuclear Force

Mass of the nucleus is

$$m_{nucleus} = Zm_p + Nm_n - \frac{B}{c^2} \approx Am_p$$

where

Z = Atomic number = number of protons: determines element

N = Neutron number: determines isotope

A = Z + N = Mass number: determines isobar

B = binding energy of nucleus (energy required to pull nucleus apart)

Nuclear density is essentially constant since nucleons are incompressible.

Strong Nuclear Force

Acts on all nucleons

Acts within 2 fm (within nucleus, 1 fm = 10^{-15} m)

Coulomb repulsion among protons

100 times weaker than Strong force

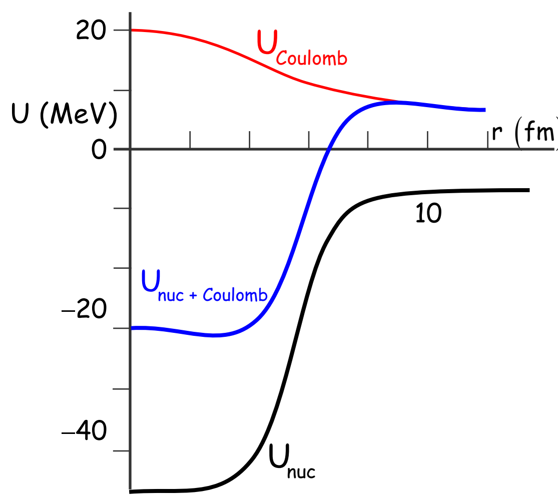
singificant only in heavy nuclei

raises energy well for p^+

Nature prefers

Tops of p^+ and n^0 wells even

Even numbers of p^+ and n^0



Masses of nuclei less than $Zm_p + Nm_n$

Mass Defect = Difference between nuclear masses

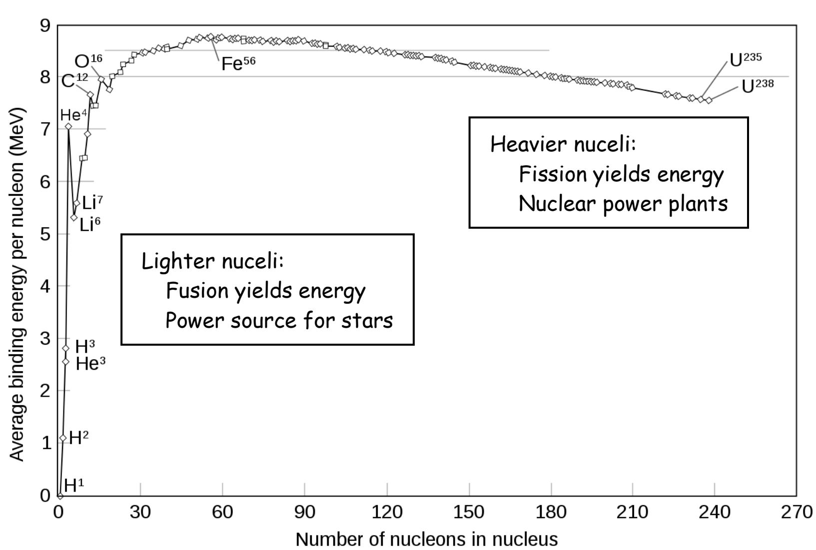
Binding energy = Mass Defect x c^2

Nucleon separation energies

$$S_n(Z,N) = B(Z,N) - B_n(Z,N-1)$$
$$S_p(Z,N) = B(Z,N) - B_n(Z-1,N)$$

Binding Energy per nucleon

Peaks at iron … $^{56}_{26}Fe_{30}$



- Radioactive Decay
  - Decay Rate

$$R(t) = rN(t)$$

    - Be able to derive the number of nuclei at a time t and an expression for the $\frac{1}{2}$ life
  - Types of Decay
    - Know the four types and their reaction equations
  - Discovery of the Neutrino
    - Why was it predicted?
    - What are the characteristics of neutrinos?
      - With what do they interact?
      - How do we know they have mass?

Nuclear Fission and Fusion

- Fission is the breaking of large nuclei into smaller nuclei
  - Produces energy for nuclei heavier than iron
  - How does it occur naturally?
  - How is it triggered in power plants and bomb?
- Fusion is the combinging of light nuclei into larger nuclei
  - Produces energy for nuclei lighter than iron
  - Where does it occur?
  - Why is it important in the evolution of the universe, Earth, life, and US?